Synthesis of Prototype Filters With Triplet Sections Starting From Source and Load

Giuseppe Macchiarella, *Member, IEEE*

***Abstract*—This paper describes a method for synthesizing an inline prototype filter with a triplet section starting from source and/or from load. This topology allows placing one or two transmission zeros without using cross-couplings inside the filter (it is only required to couple the source (or the load) to the first (or the last) two resonators. The proposed method employs the coupling matrix of a generic prototype, obtained through well-established procedures; then, the coupling matrix of the desired inline topology is determined with a procedure based on multiple matrix rotations (similarity transforms) and numerical optimization.**

***Index Terms*—Bandpass filters, circuit synthesis, elliptic filters.**

I. Introduction

It is known that the synthesis of a low-pass prototype filter with generalized Chebycheff response can be realized by various multiple-coupled topologies [1]–[3]. However, in some cases, it could be required to introduce one or two transmission zeros without using cross-couplings between resonators; this can be achieved by coupling the source (load) to both the first (last) two resonators (an example of this topology is shown in [4], concerning a dual-mode dielectric resonator filters). Recently, some works have appeared [5], [6] which treat the synthesis of a folded prototype with a cross coupling between source and load (this allows to increase the maximum number of transmission zeros up to the order of the filter). However, the inline topology described above (which can be considered as a "triplet section" starting from source or load) has not been explicitly considered in the literature (the classical method proposed by Bell [7] includes source and load, but does not give an explicit procedure for the practical implementation of the synthesis for the case here considered).

We have, then, developed a method for synthesizing the coupling matrix of an inline prototype with one or two triplet sections, starting from source and (or) load. The method starts with the synthesis of an arbitrary coupling matrix which satisfy the required filter response (the Atia ortho-normalization method [1], [3] or the Cameron canonical prototype method [2] can be employed to this purpose); then, using a suitable numerical procedure based on similarity transform, the coupling matrix of the desired topology is eventually computed.

II. Synthesis of the Prototype

It is here assumed that the prototype exhibits the generalized Chebicheff response; then, the following parameters must be given: the number N of poles, the passband return loss RL, the number and values of the imposed transmission zeros (max. 2, pure imaginary). Various methods may be found in the literature which allow to compute the characteristic polynomials determining the synthesis of the prototype (they are generally indicated as $E(s), F(s), P(s)$ [2], [3], [8]; their zeros represent respectively the poles, the reflection zeros and the transmission zeros). The scheme of the prototype that we want to synthesize is given in Fig. 1 (the meaning of the network elements is explained in the figure caption).

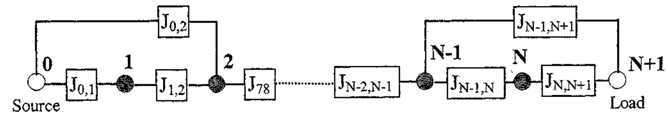

Fig. 1. Topology of the inline prototype network. Each black node represents a unit capacitance in parallel with a frequency invariant susceptance b_i; J_{ij} are the admittance inverters parameters between nodes i and j; the white nodes are the source and load resistances (assumed unitary).

In order to represent the network in Fig. 1, the formalism based on the coupling matrix can be adopted [7]

$$\mathbf{M} = \begin{array}{ccccc} 0 & J_{0,1} & J_{0,2} & \cdot & 0 \\ J_{0,1} & b_1 & J_{1,2} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & J_{N-1,N} & b_N & J_{N,N+1} \\ 0 & \cdot & J_{N-1,N+1} & J_{N,N+1} & 0. \end{array} \quad (1)$$

Note that $\mathbf{M}$ has order $N+2$ being input and output nodes included in the matrix topology.

As said before, various methods exist for synthesizing a generic prototype exhibiting the same frequency response as that here required from the network in Fig. 1 [2], [3]; however, the associated coupling matrix $\mathbf{M}'$ will have in general a very different topology from $\mathbf{M}$. Moreover, $\mathbf{M}'$ generally does not include in the topology the source and load nodes (i.e., the order of $\mathbf{M}'$ is N).

The first step for obtaining the matrix $\mathbf{M}$, once the synthesis of a generic prototype with coupling matrix $\mathbf{M}'$ has been performed, is to determine a new matrix $\mathbf{M}''$ with source and load nodes included

$$\mathbf{M}'' = \begin{array}{ccccc} 0 & J'_{0,1} & 0 & \cdot & 0 \\ J'_{0,1} & & [\mathbf{M}'] & & J'_{N,N+1} \\ 0 & \cdot & 0 & J'_{N,N+1} & 0 \end{array} \quad (2)$$

where $J'_{0,1} = \sqrt{1/R_S}, J'_{N,N+1} = \sqrt{1/R_L}$ and R_S end R_L are the source and load resistances as obtained from the synthesis of the generic prototype (note that they are in general different from unity).

Manuscript received July 10, 2001; revised November 13, 2001. The review of this letter was arranged by Associate Editor Dr. Arvind Sharma.

The author is with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Milano, Italy (e-mail: macchiar@elet.polimi.it).

Publisher Item Identifier S 1531-1309(02)01816-0.

A possible approach for synthesising the network in Fig. 1 consists in a suitable transformation of $\mathbf{M}''$ in order to obtain the same topology of $\mathbf{M}$; this can be realized by means of the matrix rotation method. This method has been widely employed for the synthesis of cross-coupled prototype filters [1], [3], [8]. Its main drawback is that the suitable sequence of matrix rotations required for given starting and ending (transformed) topologies it is not known *a priori*. Here, we propose an implementation of this method that does not require the knowledge of the optimum sequence of rotations, which is determined through a suitable optimization procedure.

It is known [3] that a similarity transform of the matrix $\mathbf{M}''$ of order n (equal to $N+2$) is defined by $\mathbf{R}_{ij}(\vartheta)\cdot\mathbf{M}''\cdot\mathbf{R}_{ij}^t(\vartheta)$, where $\mathbf{R}_{ij}(\vartheta)$ is the rotation matrix of order n, pivot (i,j) and angle ϑ, defined as follows:

$$\begin{aligned} &R_{ij}(i,i)=R_{ij}(j,j)=\cos(\vartheta),\\ &R_{ij}(i,j)=-R_{ij}(j,i)=\sin(\vartheta)\\ &R_{ij}(k,k)|_{k\neq i,j}=1, \quad R_{ij}(k,i)|_{k\neq i,j}=0,\\ &R_{ij}(j,k)|_{k\neq i,j}=0, \quad (i<j)\neq 1, \quad n. \end{aligned} \tag{3}$$

The conservation of the transfer function is also true for subsequent applications of the above transformation. So we assume that the transformed matrix $\mathbf{M_t}$ can be expressed as

$$\begin{aligned} \mathbf{M_t} &= (\mathbf{R}_{23}\cdot\mathbf{R}_{24}\cdot\cdot\mathbf{R}_{n-2,n-1})\cdot\mathbf{M}''\\ &\quad\cdot\left(\mathbf{R}_{n-2,n-1}^t\cdot\mathbf{R}_{n-3,n-2}^t\cdot\cdot\mathbf{R}_{23}^t\right)\\ &= \mathbf{S}(\vartheta_1,\vartheta_2,\ldots\vartheta_m)\cdot\mathbf{M}''\cdot\mathbf{S}^t(\vartheta_1,\vartheta_2,\ldots\vartheta_m) \end{aligned} \tag{4}$$

where m is the overall number of distinct pivots (i,j), for a given order n; m may be considered as the maximum number of independent rotations, and it is given by $m=(n^2-5n+6)/2$. It can be observed that the topology of the transformed matrix $\mathbf{M_t}$ is determined by the set of rotation angles $(\vartheta_1,\vartheta_2,\ldots\vartheta_m)$. In fact, assuming that the desired topology of the transformed matrix is compatible with the original frequency response (associated to $\mathbf{M}$), the required values of the rotation angles can be found numerically by imposing to zero the elements of $\mathbf{M_t}$ corresponding to the cross-couplings not included in the new topology. In other words, the set of rotations angles $(\vartheta_1,\vartheta_2,\ldots\vartheta_m)$ can be found by solving the following system of nonlinear equations

$$\mathbf{M}_{\mathbf{t},(k,l)}(\vartheta_1,\vartheta_2,\ldots\vartheta_m)=0 \tag{5}$$

where k and l refer to all the elements of $\mathbf{M_t}$ which must vanish (only the elements above the main diagonal are considered being the matrix symmetric).

The numerical solution of the above system may be performed through the minimization of a nonlinear cost function U defined as

$$U=\sum_{k,l}\left|\mathbf{M}_{\mathbf{t},(k,l)}(\vartheta_1,\vartheta_2,\ldots\vartheta_m)\right|^2. \tag{6}$$

In the practical implementation of the minimization procedure, the Gauss-Newton method has been used, because it allows a fast and accurate solution and it is little sensitive to the starting point.

III. Example

As an example of application of the synthesis method proposed, let consider the following electrical specifications.

- Degree 5.
- Return Loss 23 dB.
- Transmission zeros at $j1.3, j1.5$.

Using the procedure developed in [9], the polynomials $N(s), M(s), P(s)$ are computed and, from these, the generic coupling matrix $\mathbf{M}'$ from Atia [1], [3] is obtained

$$\mathbf{M}'=\begin{matrix} 0.0775 & -0.4625 & -0.7257 & -0.3809 & 0.0000\\ -0.4625 & -0.6294 & 0.5740 & 0.0344 & 0.1977\\ -0.7257 & 0.5740 & 0.2834 & -0.3849 & -0.7655\\ -0.3809 & 0.0344 & -0.3849 & -0.6603 & 0.5103\\ 0 & 0.1977 & -0.7655 & 0.5103 & 0.0775. \end{matrix}$$

The load and generator resistances have the same value: $R_{\text{gen}}=R_{\text{load}}=0.8542$.

Note that the above matrix is not unique and all its elements may be in general different from zero. Now, applying the novel procedure, the inline prototype of Fig. 1 is synthesized; the rotation angles obtained from the optimization procedure are shown in the first equation at the bottom of the page. The new coupling matrix resulting after the matrix rotations is given by the second equation shown at the bottom of the page. The computed response of the inline prototype is finally reported in Fig. 2.

$$\vartheta=[-0.3249 \quad 0.4727 \quad -0.9338 \quad -0.4478 \quad 0.5976 \quad 1.6298 \quad 0.00 \quad 0.00 \quad 0.00 \quad -2.2591]$$

$$\mathbf{M}=\begin{matrix} 0 & -0.6873 & -0.8356 & 0 & 0 & 0 & 0\\ -0.6873 & -1.1083 & 0.2326 & 0 & 0 & 0 & 0\\ -0.8356 & 0.2326 & 0.4971 & 0.7483 & 0 & 0 & 0\\ 0 & 0 & 0.7483 & 0.2257 & -0.7188 & 0 & 0\\ 0 & 0 & 0 & -0.7188 & 0.5254 & 0.4347 & 0.7025\\ 0 & 0 & 0 & 0 & 0.4347 & -0.9912 & 0.8229\\ 0 & 0 & 0 & 0 & 0.7025 & 0.8229 & 0 \end{matrix}$$

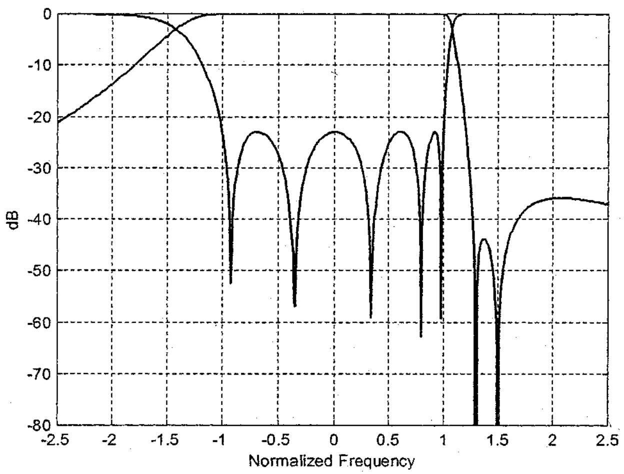

Fig. 2. Reflection and transmission response of the synthesized prototype.

IV. CONCLUSION

A procedure for the synthesis of an inline prototype filter with triplet sections including source and/or load has been presented.The prototype exhibits the generalized Chebycheff response with one or two transmission zeros placed on the imaginary axis. The proposed method is based on multiple rotations of the coupling matrix obtained from a generic prototype; the values of the rotation angles are determined through an efficient optimization procedure.

The method presented can be easily extended to accommodate also quadruplets involving source and/or load nodes (this allows to place pair of transmission zeros, either complex or pure imaginary in the prototype transfer function).

REFERENCES

[1] A. E. Atia, A. E. Williams, and R. W. Newcomb, "Narrow-band multiple-coupled cavity synthesis," *IEEE Trans. Circuits Syst.*, vol. CAS-21, no. 5, pp. 469–655, Sept. 1974.

[2] R. J. Cameron, "General prototype network synthesis methods for microwave filters," *ESA J.*, vol. 6, June 1982.

[3] ——, "General coupling matrix synthesis methods for Chebyshev filtering functions," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 433–442, Apr. 1999.

[4] I. C. Hunter, J. D. Rhodes, and V. Dassonville, "Dual-mode filters with conductor-loaded dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 2304–2311, Dec. 1999.

[5] J. R. Montejo-Garai, "Synthesis of N-even order symmetric filters with N-transmission zeros by means of source-load cross coupling," *Electron. Lett.*, vol. 36, no. 3, pp. 232–233, Feb. 2000.

[6] S. Amari, "Direct synthesis of folded symmetric resonator filters with source-load couplings," *IEEE Microwave Wireless Compon. Lett.*, vol. 11, pp. 264–266, June 2001.

[7] H. C. Bell, "Canonical asymmetric coupled-resonator filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1335–1340, Sept. 1982.

[8] G. Macchiarella and S. C. D'Oro, "Design of generalized comb filters with asymmetric transmission zeros using arbitrary cascaded triplet and quadruplet sections," presented at the *28th Eur. Microwave Conference*, Amsterdam, The Netherlands, Oct. 1998.

[9] G. Macchiarella, "An efficient technique for the synthesis of an equiripple low pass prototype filter with asymmetric frequency response and arbitrary transfer function zeros," presented at the *24th European Microwave Conference*, Bologna, Italy, Sept. 1995.